Image resolution limits resulting from mechanical vibrations. Part IV: real-time numerical calculation of optical transfer functions and experimental verification

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Abstract. A method of calculating numerically the optical transfer function appropriate to any type of image motion and vibration, including random ones, has been developed. We compare the numerical calculation method to the experimental measurement; the close agreement justifies implementation in image restoration for blurring from any type of image motion. In addition, statistics regarding the resolution as a function of relative exposure time for low-frequency vibrations involving random blur are described. An analytical approximation to the probability density function for random blur has been obtained. This can be used for the determination of target acquisition probability. A comparison of image quality is presented for three different types of motion: linear, acceleration, and high-frequency vibration for the same blur radius. The parameter considered is the power spectrum of the picture.

Subject terms: image motion; image vibration; optical transfer function; target acquisition; image restoration; robotics; modulation transfer function.


1 Introduction

Image motion is often the cause of limited resolution in reconnaissance, astronomy, robotics, machine vision, and computer vision systems. This motion degradation is generally much more severe than that from the electronics and optics. Often mechanical engineers can characterize limits of their systems stability, but optical engineers may not know how to interpret such data in terms of image quality. A convenient form with which to include such effects in system design is the optical transfer function (OTF). These functions transform the mechanical data into a format easily usable by optical engineers. Analytical OTF expressions have been developed in the literature. For uniform velocity motion, parabolic motion, and for high temporal frequency sinusoidal vibration. Other types of image motion can now be dealt with by a method developed here to calculate modulation transfer function (MTF) numerically according to modulation contrast. This paper presents a new method for calculating the OTF of image motion requiring as an input the relative function of motion between the camera and the object. This method is based on calculations in the spatial domain in contrast to previous methods in the spatial frequency domain. The advantage here is that the OTF calculation includes the phase transfer function (PTF) and is independent of a-priori information on the object. The spread function of the image motion is the probability density function (PDF) or the histogram of the image motion during the exposure time. A mathematical proof for this method is presented with comparison to known degradation functions of linear motion and high-frequency vibration.

The OTF method is not only more comprehensive than that in Ref. 3 because it also includes PTF, but is also faster. Typical calculations with an 8-yr-old 286 PC take less than 3 s. With modern PCs, an improvement of about two orders of magnitude can be expected. The limits are set by the time interval between CCD camera exposures.

Statistical behavior of MTF for low-frequency vibrations is presented here. This analysis can be used to define resolution limits from mechanical vibrations, and is also very important for target acquisition probabilities, which depend on maximum usable spatial frequency of the imaging system. This is decreased by image motion and vibration, which thus affects rather adversely target acquisition times and probabilities.

The statistical analysis is followed by experimental verification of the OTF calculation. The numerical OTF calculation for image motion and vibration is compared to that measured experimentally. The experiment was carried out with a controllable vibration platform. The object (point source or edge trace) was vibrated. These vibrations were...
converted into an electrical signal by a motion sensor. This signal was sent to a PC by means of an analog-to-digital (A/D) converter during the exposure time of the camera. By analyzing this signal with the proper software, the OTF was derived and compared to that measured via fast Fourier transform (FFT) of the image of the point source or the FFT of the derivative of the edge response. Also, the phase transfer function (PTF) can be derived numerically. Both apply to any type of motion, including random motion, in which MTF and PTF cannot be characterized analytically.

The last section includes a comparison of degradation deriving from three types of motion, linear, acceleration, and high-frequency vibration, for the same blur radius.

The OTF of image motion is the key for calculating inverse filters required for image restoration. Direct solution can be available only if the problem is suitably characterized. Some examples of image restoration based on numerical OTF calculations applied in Wiener filters derived from the method described above are presented elsewhere.4

2 Basic Method for MTF Determination

The current presentation describes briefly two new methods to calculate MTF for any type of motion, including random motion that cannot be characterized by any unique MTF.3,5

The first method is based on calculation in the spatial frequency domain.5 The second method is based on an MTF calculation carried out in the spatial domain and actually yields OTF.3 The latter method is much faster so it is more suitable for practical systems that work in real time, and it will be considered extensively. The experiment described in Sec. 4 is based on the second method. All that is required for both methods is the function of relative image motion.

2.1 Spatial Frequency Domain (MTF)

The spatial frequency domain method is based on the assumption of an object with a sinusoidal luminance pattern:

\[ i(x) = B_o + B_m \cos(2\pi fx) \quad , \tag{1} \]

where \( f \) is spatial frequency, \( x(t) \) is the motion function for spatial coordinate \( x \), and \( B_o \) and \( B_m \) are constants.

To determine the modulation of the intensity pattern of the image it is necessary to know the relative motion \( x(t) \) between the camera and the object. When this motion is known analytically, the MTF also can be obtained analytically,1 as in the cases of linear motion, high-frequency vibration, or parabolic motion. For other cases when the exact analytic function \( x(t) \) is not available, it is necessary to expand this method to numerical calculation.5 This method works with discrete points of image motion instead of the mathematical function. The MTF is calculated for each spatial frequency separately and is obtained by measuring the modulation contrast function over a multitude of closely spaced spatial frequencies.

2.2 Spatial Domain (LSF)

In this section, the line spread function (LSF) derived from image motion transverse to the optical axis is obtained. The MTF is derived as the modulus of the OTF or Fourier transform of the LSF, and the PTF is derived as the phase of the OTF. All the graphs plotted for the PTF are normalized by 2\( \pi \).

Let \( x(t) \) be the relative displacement between the object and sensor beginning at time \( t_e \) and ending at time \( t_e + t_e \), in which \( t_e \) is measured from the instant the sensor is first exposed. The LSF of the motion is the PDF or the histogram of \( x(t) \). The intuitive explanation for this determination is the following. Image motion causes the system line spread image response to move spatially. These displacements are integrated during the exposure. Such motion can be described by a histogram of the LSF, in which frequency of occurrence of a given portion \( x(t) \) is depicted as a function of \( x \) during the time interval \((t_e, t_e + t_e)\). This histogram is the LSF itself.

The quantity \( t_e \) is a random variable representing initial exposure time and is uniformly distributed according to \( f(t) = 1/t_e \).

By decomposing the relative displacement into \( n \) monotonic parts existing in one exposure time \( t_e \) that is,

\[ t_1 + t_2 + t_3 + ... + t_n = t_e \quad , \tag{2} \]

the PDF is shown to be of the form6

\[ f_x(x) = f(t) \cdot \left[ \frac{1}{1/x'(t_1)} + ... + \frac{1}{1/x'(t_n)} + ... \right] \]

\[ = \frac{1}{t_e} \cdot \left[ \frac{1}{1/x'(t_1)} + ... + \frac{1}{1/x'(t_n)} + ... \right], \tag{3} \]

where \( x' \) is the derivative of \( x(t) \) and \( f_x(x) \) is the PDF.

The lower and the upper limits, respectively, for \( x \) are the results of the minimum and maximum displacement between the object and sensor. The PDF (or histogram) \( f_x(x) \) represents the LSF. The LSF is equal to \( f_x(x) \) and the OTF is the one-dimensional (1-D) Fourier transform of the LSF:

\[ \text{OTF}(f) = \int_{-\infty}^{\infty} f_x(x) \exp(-j2\pi fx) \, dx \quad , \tag{4} \]

where \( f \) is spatial frequency.

Thus, LSF for image motion can be determined from a histogram of the motion, and the resulting OTF for such motion is given by Eq. (4).

2.3 Linear Motion OTF

If the motion is linear at a constant velocity \( V_o \) in the image plane, the resulting noncircular displacement in that direction is given by

\[ x = V_o t \quad , \tag{5} \]

where \( V_o \) is the uniform relative velocity between the object and sensor.

Hence, from Eq. (3)

\[ f_x(x) = 1/(V_o t_e) = 1/d \quad , \tag{6} \]

where \( d \) is the spatial extent of the blur and is equal to \( V_o t_e \).

Therefore, from Eq. (4),

\[ \text{MTF}(f) = \frac{1}{d} \int_{0}^{d} \exp(-j2\pi fx) \, dx \]

\[ = |\text{sinc}(\pi f d)| \exp(-j\pi f d)| = |\text{sinc}(\pi f d)| \quad , \tag{7} \]
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Fig. 1 (a) LSF, (b) MTF, and (c) PTF calculated numerically and analytically for linear motion ($d = V/2 = 1$ mm).

$$P_{TF}(f) = \text{phase}[\text{sinc}(\pi f d) \exp(-i\pi f d)] = \pi f d,$$

$$n \cdot \frac{1}{d} < f < (n + 1) \frac{1}{d},$$

$$n = 0, 1, 2, 3, \ldots.$$  \hfill (8)

Equations (7) and (8) give well-known results (Fig. 1) previously derived directly\textsuperscript{1} in the spatial frequency domain rather than in the spatial domain as shown here. Because detail of size smaller than blur radius $d$ is unresolvable, a MTF at a frequency higher than $1/d$ does not exist. A MTF at such a frequency is labeled as a false or spurious resolution.

The PTF is an additional factor shown in Fig. 1(c). The LSF given by Eq. (6) is asymmetric because it is due to motion on one side of the image.\textsuperscript{7} The result is a phase factor that is a linear function of frequency. This implies that the mean image position is shifted by $d/2$ in the positive displacement direction because of linear smear. Linear PTF is not considered a degradation process because the image can be regarded as being shifted bodily.

### 3 Image Degradation by Sinusoidal Vibration

Sinusoidal vibration is a very critical factor in dynamic imaging systems. The sinusoidal image motion is important in aircraft and vehicles because of turbines and motors that give rise to mechanical vibrations.\textsuperscript{1} In robotics and machine vision, linear motion is almost always accompanied by vibrations that are often close to being sinusoidal. The sinusoidal motion can be prevented in principle by proper design; in practice, however, it is often the most serious image motion.

Degradation of image quality as a result of sinusoidal motion\textsuperscript{8,9} depends on the ratio of exposure time $t_e$ to the period of the sinusoidal motion $T_0$. In this case, it is necessary to distinguish between two categories:

1. high-frequency vibration in which the exposure period is long compared to the period of the simple harmonic motion ($t_e > T_0$).

The LSF for this motion is given by the histogram of the sine function over one period. The exact calculation appears in Ref. 3, and the result is

$$\text{LSF}_{HF} = \frac{1}{\pi(D^2 - x^2)^{1/2}}, \quad |x| < D,$$  \hfill (9)

where $D$ is the maximum vibration amplitude. The MTF for this case is given by the Fourier transform of Eq. (9):

$$M_{TF}(f) = J_0(2\pi f D),$$  \hfill (10)

where $J_0$ is the zero-order Bessel function. The PTF for the ideal case is equal to zero because the LSF is an even function. But as is shown in Fig. 2(a), little shift occurs to the right side of the image and the LSF (analytical and numerical result). This small asymmetry causes a linear PTF between 0 and 0.4 $\pi$. The

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Fig. 2 (a) LSF, (b) MTF, and (c) PTF calculated numerically and analytically for high sinusoidal vibration frequency ($D = 1$ mm).
numerical result for the LSF, MTF, and PTF obtained by the method described in Sec. 2,2 and the analytical results are compared in Fig. 2, which shows that the graphs are similar.

It is important to mention that the result in this case remains the same for $t_e \gg T_o$. The blur radius is still the peak-to-peak displacement $2D$ as long as $t_e \geq T_o$.

2. low-frequency vibration in which the exposure period is short compared to the vibration period ($t_e < T_o$).

Quantification of the low-frequency vibrational image blur radius $d$ is much more complicated, however, because it depends on the initial phase of the oscillatory motion as well as on the instant and duration of the time exposure, both of which are often random processes. The influence of the MTF degradation is much more severe than that of the PTF; therefore, the following discussion concentrates only on the MTF.

Statistics on lucky shot probabilities and best-case and average-case MTF are detailed previously. Numerical MTF calculation in the spatial frequency domain for this case is described in Ref. 5. The most important conclusion from these references is that the MTF function is a random process.

In this section two major aspects are considered:

1. evaluation of MTF for low-frequency vibration on the basis of the LSF for high-frequency vibration

2. analysis of the (ensemble) group of the MTFs as a random process.

The method of calculating the LSF for low-frequency vibrations is based on the PDF of $x(t)$ for high-frequency vibrations in Eq. (9). Equation (9) is applied here by evaluating $f(x)$ only over the particular displacement beginning at $t_e$ and ending at $t_e + t_e$. Figure 3 schematically demonstrates the method of calculating the MTF for two extreme cases of minimum and maximum blur radius and presents a comparison to the high-frequency vibration MTF. The possibility of a lucky shot exists at the extreme points of the sine wave in which the velocity of the image is minimum. Hence, the LSF is characterized by a very narrow function so the MTF is very wide and falls off rather slowly. In the limit when $t_e/T_o$ approaches zero, the MTF tends to be constant. The other extreme case occurs around the zero crossing points of the sine wave in which the velocity of the image is maximum and quite constant. Hence, the LSF is a rectangular pulse function that leads to the MTF formulation sinc($\pi ft_{max}$) similar to Eq. (7). Mathematical expressions for minimum and maximum blur radius are defined in Ref. 1 and analytical calculation of the MTF is in Ref. 3. All the other exposures fall between these two extreme cases and, as $t_e/T_o$ increases, the variance of the MTF functions decreases so that in the extreme situation when $t_e/T_o = 1$ the MTF is the Bessel function Eq. (10).

Figure 4 presents an example of nonlinear blur radius. The LSF for this case contains two different parts of the LSF of high-frequency vibration. The first one shown at the bottom right is $f(x)$ in the interval $x(t_e + t_e) < x < x(t_e)$ and the second to the left is $2f(x)$ in the interval $x(t_e) < x < D$ because the absolute value of the derivative of $x$ appears twice in this interval. Therefore, in this case the derivative of the LSF is discontinuous in $x(t_e)$ and a jump occurs at this point. This discontinuity causes the MTF to be higher at all spatial frequencies and the MTF never reaches zero. The meaning of this result is that increasing the size of the jump in the spatial domain improves the MTF over all spatial frequencies.

This method of obtaining the OTF for low-frequency vibrations can be used in real practical imaging systems. From the known mechanical vibration frequency ($f = 1/T_o$) and the time exposure $t_e$, the OTF can be obtained for many different initial $t_e$ values in one period $T_o$. Figure 5 gives an example of using a look-up table to obtain the OTF according to the blur radius in the image plane. The advantage of this method is the simplicity of the process. Extraction of the blur radius is possible by tracking the amount of movement of a certain point in the image during two consequent exposures. For each blur radius a unique OTF exists.
3.1 MTF of Low-Frequency Vibrations as a Random Process

An important issue considered in this section is the statistical behavior of MTF functions for several different relative time exposures $t/T_o$. Let us assume a low-frequency vibration takes place in a real-time electro-optic system and the MTF is calculated by the method described in Sec. 3.2 from $t = 0$ to $T_o$ over very small intervals ($\Delta t = T_o/100$). For example, Fig. 6 shows several MTF functions for $t/T_o = 0.1$. The different MTF curves belong to different points on the time axis, that is, for each $t_o$, a MTF is associated with it. The MTF graphs were plotted only for frequencies below false resolution, that is, $f < f_{\text{max}}$. The criterion to find $f_{\text{max}}$ is shown here. The normalized MTF decreases from its value at zero spatial frequency monotonically with spatial frequency until a break point occurs; the frequency at this point was chosen to be $f_{\text{max}}$. This choice is on condition that this frequency is smaller than $1/d$, where $d$ is blur radius or LSF width. If the frequency is higher than $1/d$, then $f_{\text{max}} = 1/d$. This is consistent with the idea that $f_{\text{max}}$ cannot be smaller than actual blur radius $d$.

Figure 6 shows that the MTF is a random process (instead of time dependence, there is a spatial frequency dependence) such that each MTF can be written as $\text{MTF}(f, t_o)$, in which $t_o$ indicates the beginning of the exposure time during the vibration. The collection of all possible curves is known as the ensemble of the random process $\text{MTF}(f)$, and a waveform in this collection is a sample function of the random process.

Note that the MTF curves (sample functions) in the ensemble are not random. They are deterministic. Randomness in this situation is associated not with the MTF itself but with the uncertainty as to which MTF curve will occur in a given exposure time and the period of time vibration.

From the data in Fig. 6, using the relative-frequency measure, we can obtain the PDF for each spatial frequency. The PDF of the MTF at frequency $f$ is expressed as $P_{\text{MTF}}(\text{MTF} ; f)$. From this function the probability that the MTF will be higher than a given threshold contrast can be obtained as illustrated also in Fig. 6. A method of calculating the probability of MTF greater than the threshold $= 0.5$ is shown in Fig. 7. This probability is measured by the equation $P(f) = n1/N$, in which $n1$ is the number of points above the threshold for the specific spatial frequency $f$, and $N$ is the total number of all the sample functions (MTFs). For example, in Fig. 3, $t/T_o = 0.1$ and for $f = f_{\text{mean}} = 1/d_{\text{mean}} = 1/0.39 \equiv 2.52 \text{ cycles/mm}$, $P(2.52) = 0.244$. This method of measuring the probability can be applied for each spatial frequency as is demonstrated in Figs. 8(a), 8(b), and 8(c) for $t/T_o = 0.05, 0.1$, and $0.2$, respectively. Each graph is plotted for three threshold values: $0.3, 0.5$, and $0.7$. In Fig. 8(d), the probability results of the three relative exposures are compared for the specific threshold $0.5$.

Figure 8 shows that

1. the probability approaches zero as the spatial frequency increases. The decline of the function is very emphatic at the low spatial frequencies.
2. the probability behaves reciprocal to the threshold limit. As the threshold increases, the probability function decreases for all spatial frequencies.
3. all the graphs were plotted from zero spatial frequency up to $f_{\text{max}}$, in which $f_{\text{max}}$ is the maximum spatial frequency for specific $t/T_o$ and is equal to $1/d_{\text{min}} f_{\text{max}} = 75.1, 20.43$, and $5.23 \text{ cycles/mm}$ for $t/T_o = 0.05, 0.1$, and $0.2$, respectively, because any higher spatial frequencies would imply blur radii smaller than those that actually exist. It should be noted that $f_{\text{max}}$ is not $f_{\text{max}}$. The latter refers to MTF for a given $t_o$, while the former refers to the whole family of $t_o$ curves for a given $t/T_o$.
4. as $t/T_o$ decreases, the probability function increases [Fig. 8(d)], which is reasonable because for smaller relative time exposures degradation of the image is less severe so that the probability of obtaining higher MTF values increases.
3.2 Analytical Approximation for the Probability Function

In this section an analytical approximation to the numerical results is derived according to the principle of the minimum mean square error (MSE). Three different types of functions were considered to fit the numeric probability function: $P_1(f) = a \exp(-bf)$, $P_2(f) = a \exp(-bf) + c \exp(-df)$, $P_3(f) = a \exp(-bf) / f^c$.

The fit process was obtained for 12 different numerical data, four different relative time exposures ($t_e/T_0 = 0.05, 0.1, 0.15, \text{and} 0.2$), and three different threshold limits ($0.3, 0.5, \text{and} 0.7$). The parameter considered here is the MSE between the analytical function and the numerical data evaluated according to the method in Sec. 3.1. The analytical approximation was calculated beginning at $f_{\text{min}}$, where the probability function fell off from unity and ended at $f_{\text{cutoff}}$, where the probability function was equal to zero (Fig. 6). The best result was obtained for the third function, $P_3(f)$. The MSE for this case is minimum. The three parameters $a$, $b$, $c$, and the MSE are given in Table 1. Figure 9 presents the result for the best approximation ($t_e/T_0 = 0.2$, threshold = 0.7).

4 Experimental Setup

The purpose of the experimental setup is to compare LSF measurements describing the degradation of a vibrated object.
with the LSF calculated numerically by the method in Sec. 2.2 from displacement of that same source.

Figure 10 presents the main setup used for these experiments. The experiment is carried out with a controllable vibration table (shaker) that moves back and forth in a horizontal sinusoidal motion. A point light source or an edge trace was attached to the shaker. The shaker is limited to vibrational frequencies up to 4 Hz. Amplitude of motion is 50 mm, peak to peak. The relative time exposure can be controlled by changing the exposure CCD integration time. This makes it possible to observe results for both categories of sinusoidal vibration: low- and high-frequency vibrations. These vibrations are converted into an electric signal by a motion sensor. This signal is sent to the computer through an A/D converter during the exposure time of the camera. The analysis to determine the degradation functions LSF and MTF are carried out in real time with proper software.

4.1 Light Point Source

Actual light point sources are easy to visualize and convenient in mathematical analyses, but are difficult to realize in prac-

**Table 1 Analytical approximation for the probability density function:**

\[ P(f) = a \exp(-b \cdot f) / f^c. \]

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<th>( L_0 )</th>
<th>Threshold</th>
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<th>( b )</th>
<th>( c )</th>
<th>MSE</th>
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**Fig. 8** Probability that the MTF is greater than the threshold = 0.3, 0.5, and 0.7 for (a) \( t_0 / T_0 = 0.05 \), (b) \( t_0 / T_0 = 0.1 \), (c) \( t_0 / T_0 = 0.2 \), and (d) \( t_0 / T_0 = 0.05, 0.1, 0.2 \), and a threshold = 0.5.

**Fig. 9** Analytical approximation for the probability function, \( t_0 / T_0 = 0.2 \) and a threshold = 0.7.

**Fig. 10** Experimental setup.
tice. However, for practical purposes, a source of finite dimensions is a point source if its dimensions are negligibly small compared to other significant dimensions in the optical configuration. In the case of a digitized image, the image of the point source should be smaller than a pixel. In this experiment the light source is a low-power halogen lamp light covered by a pinhole 0.3 mm in diameter. The size of a picture element referred to the object plane was 0.5 mm. Using a pinhole that is very narrow compared to a single pixel sustains the condition of a point source; however, the radiation observed by a CCD camera is then too small to detect blurring. The most important requirement for this source is to obtain uniformly radiant intensity in different directions. To achieve uniform intensity, the point source must be wide so it will be less sensitive to misalignment of the camera lens and the light source axis. This condition is especially important here because the light point source is vibrated against the CCD camera. To obtain the actual LSF of the blur, radiation observed by the camera from different angles must be the same.

An alternative object can be a passive image of an edge trace that is less sensitive to radiation from different angles if background illumination is fairly uniform.

An edge trace is another important tool for determining the degradation function. As is known from linear systems, the system impulse response is equal to the derivative of the step response function. The step function in optics is the edge trace or the knife edge. The edge trace is the integral of the LSF.

![Diagram](image1)

**Fig. 11** Obtaining the degradation functions from edge trace: (a) still image, (b) linear motion, and (c) nonlinear motion.

### 4.2 Edge Trace

For an ideal optical system, the resulting image of an edge trace is a half "white" and half "black" plane separated by a straight line, which is the image of the knife edge. Ideally, the resulting LSF for this system is the delta function so that the MTF is constant [Fig. 11(a)]. In practice, the transition from white to black in the image is not abrupt but occurs gradually because of nonideal optics, diffraction, aberrations, defocus, and environmental problems such as mechanical vibrations. In our experiment, the dominant degradation process is caused by the image vibration.

The advantage of this passive method is the nonsensitivity to radiant intensity in different directions. The results obtained here are somewhat better than those of the experiment with the point light source.

### 4.3 Camera

A Pulnix CCD camera was used in this experiment along with a Fuji zoom optical lens. The camera has a fixed exposure time of 20 ms over which the incoming light is integrated. The CCD is 768 (horizontal) × 493 (vertical) pixels. Each pixel integrates the incoming light for 20 ms. During the next 20 ms, the data is shifted into a buffer and written out serially into the communication channel. One other important aspect of the CCD array is that always one field (half of the whole array) is exposed simultaneously, so each pixel in its proper field is exposed to the same motion. The video
mode system described above is an interlace mode that is not appropriate for this experiment for two reasons:

1. Because of relative motion between the camera and the object during the exposure time, the two different fields are not exposed at the same time. This causes the blur on the image monitor not to be the real blur of the image.

2. In interlace mode the integration time is constant (20 ms for each field) so to control the relative time exposure ($t_e/T_o$) by the parameter $t_e$ is not possible. The only way to change $t_e/T_o$ is to control the vibration time period that is limited to only 4 Hz, therefore, $t_e/T_o$ is limited to 0.08.

The solution for this experiment is to work in the integration mode. In this mode it is possible to work with full integration over the entire CCD array, and time integration is controlled by pulses from the computer. The advantage of operating in this mode is that the minimum integration time is 35 ms for the whole array and it is possible to choose any exposure time larger than 35 ms. The requirement in this mode is to supply the sync pulses (horizontal and vertical) from outside the camera by a sync generator. In this experiment the SAA1043 model sync generator is used for this purpose.

### 4.4 Motion Sensor

The motion sensor is a model S-500V vibration transducer that detects the instantaneous value of the vibration velocity, which can be used in the horizontal and vertical directions. The operating frequency range lies between 1 and 1000 Hz with a maximum detectable vibration velocity of 500 mm s$^{-1}$. The vibration mass consists of a coil that moves in the field of a permanent magnet and thus generates a voltage proportional to the vibration velocity. The output signal is 1 V per 50 mm s$^{-1}$ and its maximum value is $\pm$ 10 V.

### 4.5 Experimental LSF Measurement

The experiment is divided into two parts that take place simultaneously as shown in Fig. 12. The first part involves the LSF measurement from the image and the second calculates the LSF from the signal of the movement sensor. For an ideal object (point source of an edge trace) and measurement system, both LSF results should be identical. The purpose of measuring both is to justify depending eventually on the motion sensor alone. The experimental sequence is as follows:

1. The object is imaged continuously with a video camera and focused so that image of the light point source is less than 1 pixel on the image monitor. The amount of light incoming into the camera is controlled so the image is not saturated.

2. The field of view of a picture element is determined by taking a frozen picture of a rule and finding the number of pixels it occupies in the image plane. This task is very important for scaling the system so that it can be possible later on to compare LSF from the image and sensor.

3. The time integration $t_e$ option is chosen.

4. The camera now works in a single-frame mode, and the data of the vibrated image is transferred to the image processing unit (MTRWOX card). The data from the motion sensor is sent continuously to the computer.

5. The desirable picture area is chosen. This area does not include the entire CCD array but only a small portion of it (40 x 150 pixels) around the blurred image in the case of the point light source and around the border between the white and black in the edge trace. At this time the A/D converter digitizes the image motion for the exact time of the exposure.

6. The first mathematical processing is performed on the sensor signal with C language. This includes the integration of the velocity to obtain the displacement of the image of the object.

7. All the other processing calculations are performed with Matlab software. These include:
   a. numerical histogram of the displacement to determine the LSF according to image motion
   b. point light source response in which the picture of the blurred image along the columns is averaged to obtain a 1-D PSF, which is the LSF from the picture
   c. edge trace, in which the process of obtaining the LSF function is a little different from the case of the point light source: (1) find the center of the window (40 x 150 pixels) including the edge trace (the algorithm is based on finding the maximum derivative of one row to find the center position in the x
Fig. 13 Example of low-frequency vibration ($t_e/T_0 = 0.8$) equal to the experimental results: (a) displacement, (b) LSF comparison between image and sensor, (c) MTF comparison, and (d) PTF comparison.

axis); (2) average the image in the accepted window—column averaging; (3) operate a nonlinear smoothing on the averaged gray level; and (4) find the derivative of the accepted vector to obtain the LSF function. Figure 11 presents the process to obtain the LSF from the edge trace for three different cases with the same size blur radius $d$: (a) the still image, in which the LSF is the delta function and the MTF is constant; (b) linear blur radius, in which the gray-level function is linear with displacement and the derivative is constant in which case the MTF is the sinc function as in Eq. (7); and (c) nonlinear blur radius, in which the LSF is also nonlinear so the MTF obtains a higher value.

d. comparison between both LSF functions, image against sensor

e. Fourier transformation of the LSF to obtain MTF and comparison of the experimental results to the analytical one when the latter exists, such as $J_0(2\pi fD)$ for high-frequency vibration.

Fig. 14 High-frequency vibration ($t_e/T_0 = 4$) experimental results: (a) displacement, (b) LSF comparison between image and sensor, (c) MTF comparison between image sensor and the analytical function—$J_0(2\pi fD)$, and (d) PTF comparison.

The results of this experiment with the point light source for two representative cases are shown in Figs. 13 and 14, and one example with the edge trace is shown in Fig. 15. In Fig. 13, $t_e/T_0 = 0.8$, which represents the low-frequency vibration case. In Fig. 13, the blur shape is different from linear motion and the blur radius is 25.55 mm. Figure 14 represents the case of high-frequency vibration $t_e/T_0 = 4$, and the blur radius is the peak-to-peak relative displacement $2D = 30.48$ mm.

Figure 15 describes the results obtained by edge tracing for $t_e/T_0 = 0.67$. Figure 15(a) presents the target motion as measured by a motion sensor, Fig. 15(b) is the image motion edge response, Fig. 15(c) presents the sensor and image LSFs, and Fig. 15(d) presents the sensor and image MTFs. These four graphs are only a few examples out of many.

Several conclusions can be drawn from these experiments:

1. In the case of low-frequency vibration (Fig. 13), the agreement between the LSF from the image and the LSF from the motion sensor is better than the result in the high-frequency case (Fig. 14). The reason for this...
is connected with the relatively long exposure time. The amount of noise that enters the system is much higher: more background noise exists in the image, and more electronic noise exits from the sensor.

2. The MTF is calculated by the Fourier transform of the LSF, and the agreement here (image against sensor) is better than that of LSF comparison. The reason for this is that the FFT integration operation smooths out the differences between the two MTFs.

3. LSF measurement from the image is very sensitive to the nonhomogeneous nature of the point light source, as mentioned above. The solution for this problem is to widen the radiation angle of the point source.

4. The PTF presented in Fig. 13(d) is nonlinear with the spatial frequency as in the case of linear motion [Fig. 1(c)] and, therefore, the phase distortion is not negligible. For the case of high-frequency vibration [Fig. 14(d)], the phase function can be approximated to a linear function so the distortion is less severe.

5. Comparing the edge trace results to the light point source results shows an improvement of the measured blur radius. On the other hand, the SNR by the edge method is lower because it involves a reflective passive light pattern.

5 Degradation Process Comparison Between Three Types of Motion

A comparison of image quality is presented here for three different types of motion: linear, acceleration, and high-frequency vibration for the same amount of blur radius. The parameter considered is the power spectrum of the image.

An investigation of acceleration motion compared to constant velocity (linear motion) reveals a surprising result, as in Ref. 6. For the same extent of smear, the modulus of the transfer function caused by uniformly accelerated relative motion is, at all spatial frequencies, equal to or greater than the modulus of the transfer function caused by uniform relative motion. The MTF of accelerated motion for the same blur radius improves with an increase of acceleration or a decrease of the parameter \( r = \frac{v^2}{a} \), where \( v \) is constant velocity and \( a \) is constant acceleration. As \( r \) decreases, the MTF for the case of acceleration becomes higher at all spatial frequencies so that the image degradation process is less severe.

For the case of high-frequency vibration, the amplitude of the vibration is peak-to-peak displacement \( d = 2D \). In this case, the MTF is the well-known zero-order Bessel function \( J_0(2\pi fD) \). The first zero of this function occurs at \( f_{\text{max}} = \frac{0.7655}{d} \), compared to the case of linear motion in which \( f_{\text{max}} = 1d \). Note that the width of the MTF for the case of acceleration motion is determined according to the criterion to obtain \( f_{\text{max}} \) because MTF for this case never reaches zero. Therefore, the width of the MTF is smaller than in the cases of linear motion or acceleration motion.

Figure 16, a block diagram of the system, shows source image transfers through three synthetic filters for three types of motion. The next operation is a comparison between the power spectrum of the source image and the degradation spectrum.

Figure 17 shows the result of this simulation for the case of \( d = 15 \text{ mm} \) and \( r = 0.01 \) for the acceleration motion. As is
The statistical behavior of the MTF function is very useful in the case in which the duration of the relative motion between the camera and the object is long compared to the exposure time such as in low-frequency vibration. For example, this analysis is useful also for an imaging system mounted on a tank that will suffer from impulse motion during firing. The MTF in this case is also a random process because it depends on the initial time exposure during the impulse response motion. This statistical result can also be used for defining the probability of detecting a target in an image taken during a vibration period by a human visual system and accounting for the integration time of the eye.

The comparison of the degradation process presented in Sec. 5 is important in the design of target acquisition systems. The identification of the target is easier in the case of acceleration motion compared to linear motion or high-frequency vibration for constant blur radius.

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